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Using Objective Measurements of Plant and Soil Characteristics to Forecast Weight of Grain Per Head for Winter Wheat

by

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ABSTRACT

Techniques to improve the models currently used to forecast weight of grain per head for winter wheat have been developed. Variables now used in the present model are acceptable; however, a new variable, length of head, could be used in place of the number of fertile spikelets. The correlation of length of head with weight of grain per head in the two states studied is almost as high, and the cost of obtaining the measurement would be much less than obtaining a count of fertile spikelets. None of the soil characteristics measured were significantly correlated with the weight of grain produced per head. Regression coefficients as now computed for use in the forecast model are biased downward. The bias arises from including within field sampling errors in the data used for the calculations.

INTRODUCTION

The predicted average weight of grain per head is the major source of forecast error in the winter wheat forecast-model. In Kansas, for example, the relative (to the mean) forecast errors for weight of grain per head in 1969 were more than twice as large as for the forecast number of heads.

The original purpose of this study was to investigate the possible use of other vegetative characteristics in an early season wheat forecast model to predict weight of grain. This led to the basic problem as to how the regression coefficients for the forecast models should be computed.

SRS prepares monthly estimates of the expected yield of winter wheat, from May through October for 24 of the 48 conterminous states. These estimates are released to the public about the tenth day of each month. The estimates are generally based upon surveys taken the last week of the preceeding month although a few states also use data from soil moisture surveys. Several states also use monthly precipitation in a multiple-regression forecast model. The majority of the monthly surveys are non-probability samples and require the respondent to make some type of subjective evaluation of the prospective wheat crop on his farm or in his locality. However, forecasts based on a system of objective observation on plants in a probability sample of wheat fields have been used in the 17 major wheat states for several years. Due to restricted computer capabilities at the time the wheat objective yield models and computer program were developed, the objective yield forecast weight of grain per head for a particular sample is based on maximum of two observed variables in separate simple linear regression prediction equations. Furthermore, each of the two variables is used with two separate sets of coefficients. One set of coefficients for a particular variable is computed from historic data from the particular state. The second set of coefficients is computed from historic data from several states in that region. The computer program which summarizes the objective data and prepares the four individual estimates also computes a weighted forecast average weight of grain per head. The weights used here are inversely proportional to the forecast errors observed in previous years.

No attempt is made to predict the weight of grain per head until the sample is in at least category 3, "Late Boot or Flower." Observed values (X's) used in the prediction equations for categories 3, 4, 5 and 6 are state office laboratory determinations of

a) the average number of fertile spikelets per head, category 3 only,

b) a count of grains per head, categories 4, 5 and 6 and

c) the average gross (unthreshed) head weight, categories 3 through 6.

A more complete description of the wheat forecasting model is in the Objective Yield Supervising and Editing Manual, Section 15A, "Wheat Forecasting and Estimating Models."

DATA COLLECTION

1967

Data was collected from eight fields in each of two states, Oklahoma and Montana. The fields in each state were purposely selected to represent a variety of conditions and yields. Field observations were to have started before the field had reached maturity category 2, "Flag or Early Boot," but the fields in Oklahoma were already past this stage before the project started.

Three sample units, each 5 rows wide and 30 feet long were located randomly within each field. Several count sections, each with adjacent clip sections, were staked out in each unit. Count section 1 was used for the first two weeks of the study, then count section 2 for the next 2 weeks, then count section 3, etc. until harvest. Rows 1, 3, and 5 in the sample units were used as buffer areas to reduce the effect of previous observations on the sample plants. Buffer areas were also left between the adjacent count-clip sections. Plants in the clip sections were clipped each week until harvest--a different clip section each week - starting when the flag leaves first appeared. These plants were clipped at the ground level and sent to the state office for further observations. The enumerator also picked ten flag leaves from outside the unit. These leaves were taped to cardboard and also sent to the state office.

Soil samples were taken from each field on alternate weeks, starting with the first week. The samples were taken to a depth of 4 feet.

1968

Research in 1968 was conducted on a twenty percent subsample of the regular objective yield sample fields in Montana only. Any of the subsample fields selected which were not to be harvested as winter wheat for grain were replaced by the next lower numbered sample. All observations were taken at monthly intervals, at the same time as the regular objective yield survey.

A single research unit was located in each field halfway between the regular objective yield sample plots. The research unit was a single one-foot count section and three one-foot clip sections. The first ten plants in the count section were tagged for monthly flag leaf measurements. A different clip section was used each month. Flag leaves from the first ten stalks in the clip section were measured, clipped from the plant, and mailed to the SSO for laboratory determinations. A soil sample was also taken each month next to the research unit.

THE FORECAST MODEL

The linear regression model used in predicting the weight of grain per head (Y_i) assumes that a particular Y_i can be described mathematically as

$$Y_i = a + bX_i + e_i$$

The X_i is a statistic observed on plants which are associated with the ith sample, and a and b are parameters estimated from data obtained in previous years. The statistic e_i represents the difference between the predicted and actual values of Y. That is if:

$$Y_i = a + bX_i, \text{ then } e_i = Y_i - Y_i$$

The e's are assumed to be random variables from a single population with mean zero and variance,

$$\sigma_e^2 \text{ (or } \sigma_{y.x}^2\text{)}$$

The variance may be computed directly from the e_i's if known, or it may be estimated as

$$\sigma_{y.x}^2 = \frac{\Sigma y^2 (1 - r^2)}{n - 2}$$

Where: Σy^2 is the sums of squares of the Y_i adjusted for the average final head weight.

r is the coefficient of correlation between the X_i and Y_i, and

n is the number of observations in the sample

The precision of the estimate is inversely related to σ_e^2 . Therefore, to reduce σ_e^2 , one can increase n or increase r. Increasing n makes the survey cost larger. Increasing r can be done by finding new and better variables or by producing better estimates of the regression coefficients.

ANALYSIS

Plant Characteristics

Specific plant characteristics studied were:

(1) Flag leaf length and width,
(2) wheat head length,
(3) number of fertile spikelets per head,
(4) number of kernels per head.

Relationships of average plant characteristics were studied rather than relationships of individual plants. That is, the length of the flag leaf from plant A in sample one was not correlated with the weight of grain produced by plant A (measured at harvest). Instead, the correlation was between the average flag leaf length from a plot and the average production of grain per head from the same plot. Since a random selection of plots was made, the associated errors are independent. Independent errors are necessary for regression theory to apply.

The data collected in 1968 came from twenty fields. Eight of these fields were the Cheyenne variety, eight fields were the Winata variety and four fields were other varieties.

The data was analyzed so that if one variety had a different regression line from another, the difference could be detected.

Unthreshed head weight was used as the dependent variable. This variable can be adjusted by a factor of about 2/3 (depending on moisture) to estimate the threshed weight of grain.

(1) Length of Flag Leaf

One of the principal objectives of this study was to evaluate the possible use of flag leaf measurements for forecasting final head weight. Research in Canada had indicated that this plant characteristic might be correlated with production of grain per head. In 1967, flag leaves were clipped and immediately fastened to cardboard with transparent tape. The tape was applied so that the leaf was completely covered with tape. Moisture in the leaves could not pass through the tape very readily but could be absorbed by and pass through the cardboard. In an effort to determine how long it would take the flag leaves to dry (and shrink) down to a constant area, the cards of flag leaves were duplicated as they were received at the state offices and at weekly intervals thereafter. Duplicating machines used were a Xerox copier in Oklahoma and a Thermofax in Montana. The areas of the original leaves and of the copies were determined by planimetering.

As planimetering progressed, we found that many of the copies had smaller areas than did the original dried out leaves. After an intensive investigation, we found that the duplicating machine made exact copies only if the intensity control of the copier was at a particularly high setting. Partially because of this factor and also because of the amount of time, i.e., cost, required to planimeter the leaf area, other functions of leaf size, e.g., length and weight, were studied rather than leaf area.

While both of these characteristics were found to be highly correlated with flag leaf area, the correlation of length with area was much higher than was that of weight with area (.95 vs. 51). Since the leaf length can also be obtained without destroying the plant, further studies were limited to this one factor.

One objective in 1968 was to find the relationship between final head weight and flag leaf length. To study the relationship, a test is needed to determine whether or not different varieties have a homogeneous relationship. That is, the forecast parameters for variety I

$$(\hat{Y}_{1j} = a_1 + b_1 X_{ij})$$

might need to be different from the parameters of variety II

$$(\hat{Y}_{2j} = a_2 + b_2 X_{2j}), \qquad \text{etc.}$$

A sequential test was used which first tested the equality of the regression coefficients or slopes ($b_1 = b_2 = b_3$). 1/

If the slopes are equal, then the regression intercepts are tested to see if they are equal ($a_1 = a_2 = a_3$), etc.

If the regression slopes and intercepts are equal, then the regression coefficient is tested to see if it is significantly different from zero. That is, is the b enough different from zero so that the x-variable is useful in forecasting the final head weight.

These tests are illustrated in Table 1.

The first test is to determine if the regression slopes are equal or not. The F-value of 1.058 is not significant; therefore, the slopes are assumed

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1/ This sequential test is explained in the report "A Study of New Objective Yield Procedures for Filberts," William H. Wigton, Research and Development Branch, Standards and Research Division, Statistical Reporting Service, March, 1970.

Table 1.--An analysis of variance testing the suitability of regression coefficients, flag leaf length versus unthreshed head weight for different variety groups, Montana, 1968.

Sources of variation	Degrees of freedom	Sum of squares	Mean squares	F-test	Hypotheses
Between variety groups	2	.095028	.0475	.808	H_o: $Y_i = Y_j$
Within varieties	17	.999852	.0588		H_a: $Y_i \neq Y_j$
Total	19	1.094880			
Regression	1	.01115	.01115	.185	H_o: $Y_{ij} = \bar{Y}$
Error 1	18	1.08373	.06021		H_a: $Y_{ij} = a + bX_{ij}$
Between intercept values	2	.08392	.04196	.671	H_o: $Y_{ij} = a + bX_{ij}$
Error 2	16	.99981	.06249		H_a: $Y_{ij} = a_i + bX_{ij}$
Between regression coefficients	2	.13131	.06566	1.058	H_o: $Y_{ij} = a_i + bX_{ij}$
Error 3	14	.86850	.06204		H_a: $Y_{ij} = a_i + b_iX_{ij}$

to be equal. The second test is to determine whether the intercepts are significantly different from each other. The smaller F-value of .671 is not enough to reject the hypotheses that

$$a_1 = a_2 = a_3.$$

The next test was to determine if the regression coefficient was different from zero. If not, then the length of flag leaf would be of no value in obtaining the estimate. The F-value is .185. The square of the correlation coefficient of .01 is too small for the flag leaf length to improve forecasts of final head weight. The data in this table then indicates that flag leaf length is of no value in estimating weight of grain per head.

(2) Head Length

The length of the wheat head was also evaluated as a possible x-variable in a regression estimator. While the data for this report was collected at harvest, it was assumed that the wheat head reaches its final or maximum head length at some earlier maturity category. At what maturity level, head length might be used to estimate the final head weight? The same sequence of tests were used to find the relationship between head length and head weight (Table 2).

The first test, starting at the bottom, ($b_1 = b_2 = b_3$) is not significant.

The slopes may be assumed to be equal. The second test which tests whether or not the intercepts are equal ($a_1 = a_2 = a_3$) has an F-value of 2.33. It

is significant at the eighty percent level, but not at the ninety-five percent level. The last test considered was whether $b = 0$. The F-value for this test was highly significant. We reject the hypothesis that $b = 0$.

The results in Table 2, show (1) the b in the model $Y = a + bX_i$ is the same for all the varieties, but (2) the a's may be different. Before this variable could be used, a more complete study should be made to determine if there are differences in the average head length between the various varieties.

(3) Number of fertile spikelets per head

The number of fertile spikelets is used in the present estimation model for maturity category 3. The following analysis again is based on data collected at harvest. It could differ slightly from data collected in maturity category 3 because of counting errors and changes in plant characteristics. In this analysis it has been assumed that a fertile spikelet in category 3 should be fertile in category 6 and vice versa. Table 3 shows the tests for the regression coefficients of the fitted least squares line of the number of fertile spikelets versus head weight.

Table 2.--An analysis of variance testing the suitability of regression coefficients, average length of head versus unthreshed head weight for different variety groups, Montana, 1968.

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F-test	Hypotheses	r^2
Between variety groups	2	.07704	.03852	.68	$H_o: \bar{Y}_i = \bar{Y}_j$	
Within varieties	17	.96257	.05662		$H_a: \bar{Y}_i \neq \bar{Y}_j$	
Total	19	1.03960	.05472			
Regression	1	.22372	.22372	4.94 1/	$H_o: Y = \bar{Y}$	.22
Error 1	18	.81588	.04533		$H_a: Y = a + bX$	
Between intercept values	2	.18424	.09212	2.33 2/	$H_o: Y = a + bX$	.34
Error 2	16	.63164	.03948		$H_a: Y = a_i + bX$	
Between regression coefficients	2	.00660	.00330	0.07	$H_o: Y = a_i + bX$	.35
Error 3	14	.62504	.04465		$H_a: Y = a_i + b_iX$	

1/ Significant at the 95 percent level

2/ Significant at the 80 percent level

Table 3.--An analysis of variance testing the suitability of regression coefficients, fertile spikelets per head versus unthreshed head weight by variety groups, Montana, 1968.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F-test	Hypotheses	r^2
Between variety groups	2	.07704	.03852	.68	Ho: $\bar{Y}_i = \bar{Y}_j$	
Within varieties	17	.96257	.05662		Ha: $\bar{Y}_i = \bar{Y}_j$	
Total	19	1.03960				
Regression	1	.29046	.29046	6.98 1/	Ho: $Y_{ij} = \bar{Y}$	.28
Error 1	18	.74914	.04162		Ha: $Y_{ij} = a + bX_{ij}$	
Between intercept values	2	.04804	.02402	.55	Ho: $Y_{ij} = a + bX_{ij}$	.27
Error 2	16	.70110	.04382		Ha: $Y_{ij} = a_i + bX_{ij}$	
Between regression coefficients	2	.05272	.02636	.57	Ho: $Y_{ij} = a_i + bX_{ij}$	.33
Error 3	14	.64839	.04631		Ha: $Y_{ij} = a_i + b_iX_{ij}$	

1/ Significant at 95 percent level

The results in Table 3 indicate it is not necessary to use separate models for varieties. One regression coefficient and intercept can be used for all varieties. The r^2 of .28 means that the sum of squared deviations from the regression line is 28 percent smaller than the sum of the squared deviations from the mean. One problem is that a count of the number of fertile spikelets on a plant is time consuming. A possible substitute might be length of head. The difference in the r^2's of these two variables (fertile spikelets, r^2 = .28, length of head, r^2 = .22) was tested. However, neither variable is sufficiently correlated with weight to be a very satisfactory variable in the model. Since we assume that the underlying population correlations are not zero, the distributions of these values (r^2's) are not normal. The test uses Fisher's Z-transformation to change the non-normal r-values to Z values which are distributed approximately normmally and with variance $1/(n-3)$.

$r_1 = \sqrt{.28} = .529$ $\qquad r_2 = \sqrt{.22} = .469$

$Z_1 = .59$ $\qquad Z_2 = .51$

$n_1 = n_2 = 20$ observations. The variance of Z_1 = variance of $Z_2 = \frac{1}{17} = .059$

$$S.E. (r_1 - r_2) = \sqrt{.059 + .059} = .344$$

$$Z = \frac{.590 - .510}{.344} = .23$$

The computed value of .23 is much too small to warrant rejecting the hypothesis that the coefficient of correlation for fertile spikelets (ρ_1) is equal to the coefficient of correlation for head length (ρ_2). Therefore, the average length of wheat head could be used instead of the average number of fertile spikelets with a considerable reduction in cost, and no apparent loss in precision.

(4) Number of kernels per head

The average number of kernels per head is used in the present forecast model for maturity categories 4, 5, and 6. A F-test was used to compare the regression coefficients for the different variety groups. (Table 4)

This set of test indicates that the same regression coefficient and intercept can be used for all varieties without appreciable loss in efficiency. The correlation of this variable with the gross head weight is the highest of all the variables studied in this report.

Table 4.--An analysis of variance testing the suitability of regression coefficients, average number of kernels per head versus unthreshed head weight by variety groups, Montana, 1968.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F-test	Hypotheses	r^2
Between variety groups	2	.07704	.03852	.68	Ho: $\bar{Y}_i = \bar{Y}_j$	
Within varieties	17	.96257	.05662		Ha: $\bar{Y}_i \neq \bar{Y}_j$	
Total	19	1.03960	.05472			
Regression	1	.66745	.66745	32.28 1/	Ho: $Y_{ij} = \bar{Y}$	.64
Error 1	18	.37215	.02068		Ha: $Y_{ij} = a + bX_{ij}$	
Between intercept values	2	.06900	.03450	1.82	Ho: $Y_{ij} = a + bX_{ij}$	.69
Error 2	16	.30316	.01895		Ha: $Y_{ij} = a_i + bX_{ij}$	
Between regression coefficients	2	.02002	.01001	.49	Ho: $Y_{ij} = a_i + bX_{ij}$	.71
Error 3	14	.28314	.02022		Ha; $Y_{ij} = a_i + b_iX_{ij}$	

1/ Significant at 99 percent level.

Soil Characteristics

The soil characteristics studied showed virtually no relationship to final head weight. None of the computed correlations (Table 5) were significantly different from zero.

The r 's are quite low; the highest one, percent silt, reduces the total variation by only 17 percent. A stepwise multiple regression program was used to select the best variables out of a set of variables. When all the plant characteristics and soil characteristics are placed together and analyzed using this program, none of the soil characteristics get selected in the first three places. This is reasonable, since the individual correlations are low and any observed plant characteristic would surely include some of the effect of the soil characteristics.

Table 5.--Coefficient of correlation between various soil characteristics and average final head weights, Oklahoma and Montana, 1967 and 1968.

Soil Characteristics	Degrees of freedom	Coefficient of correlation
Percent of silt	18	.41
Percent sand	18	.30
pH	18	.15
Phosphate	18	.13
Organic matter	18	.08
Potash	18	.05
Percent moisture	18	.03
Percent clay	18	.02

Regression coefficients by the method of least squares

This report has been based on a regression estimator of the type $Y_i + e_i = a + bX_i$. If this model is to provide good estimates, the following underlying assumptions are required:

(1) The X's and Y's must be paired values from a bivariate normal distribution, and e's be normally distributed with mean zero, and be independent of the value of X and Y.

(2) The variance of Y must be the same at every X, i.e.,

$$\sigma^2 Y\ X_i = \sigma^2 Y\ X_j = \sigma^2 e.$$

(3) The X's must be measured without error.

The validity of these assumptions depends on the underlying distributions of the populations and the procedures followed in collecting the data. It seems reasonable to assume that the distribution of the population values is a bivariate normal. Random samples of data are selected, so it follows that the e's are normally distributed and independent of the values of X. Assumption two requires that the conditional variance of Y given X_1 (σ^2 y X_i), and Y given X_2 (σ^2 y X_i), etc., to be equal. These conditional variances could be computed and tested to see if they were equal. It seems reasonable that they would be nearly equal.

The third assumption requires that the X's be measured without error (sampling or experimental). Nothing is measured without some error, but gross errors are of concern. To evaluate the size of the measurement errors, the field and laboratory procedures and equipment used to obtain the observation values that must be studied. The field procedures used in 1967 were different from those used in 1968. It is worth digressing for a moment to look at the two field procedures and their effects.

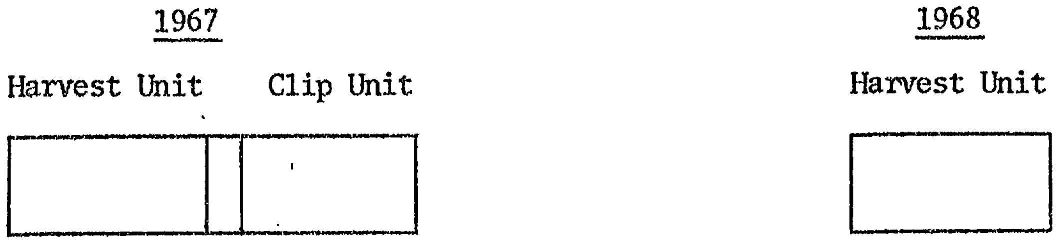
In 1967, destructive sampling was used. A sample plot was selected at random and divided into units. The first month one unit was clipped and the sample was taken to the state laboratory. Finally, at harvest, still another unit was clipped and sent to the laboratory. The Y's (harvest values) were paired with the X's (early season values) and a regression equation was developed. This introduced both sampling errors (between unit variation) and experimental error due to variations in measurements. Unless all plots are exactly alike sampling errors will be introduced whenever the X and Y values are taken from different plots, this must occur when destructive sampling is used to obtain the X values.

In 1968, observations were taken on the same plants in one unit throughout the growing season. At harvest, the unit was clipped and sent to the state laboratory. All X and Y values came from the same plants. No between plot sampling errors were introduced since the X values from one unit were not substituted for X values of another unit. The two sampling schemes are illustrated in Figure 1.

One must consider what happens when sampling errors are introduced in the data collection procedures. The regression coefficient (b) can be defined as the covariance of X and Y divided by the variance of X.

$$b = \frac{\text{Cov (XY)}}{\text{Var (X)}}$$

Figure 1.--Comparison of sampling procedures in 1967 and in 1968.



Y = Average head weight in harvest plot

X = Average number of kernels per head in harvest plot

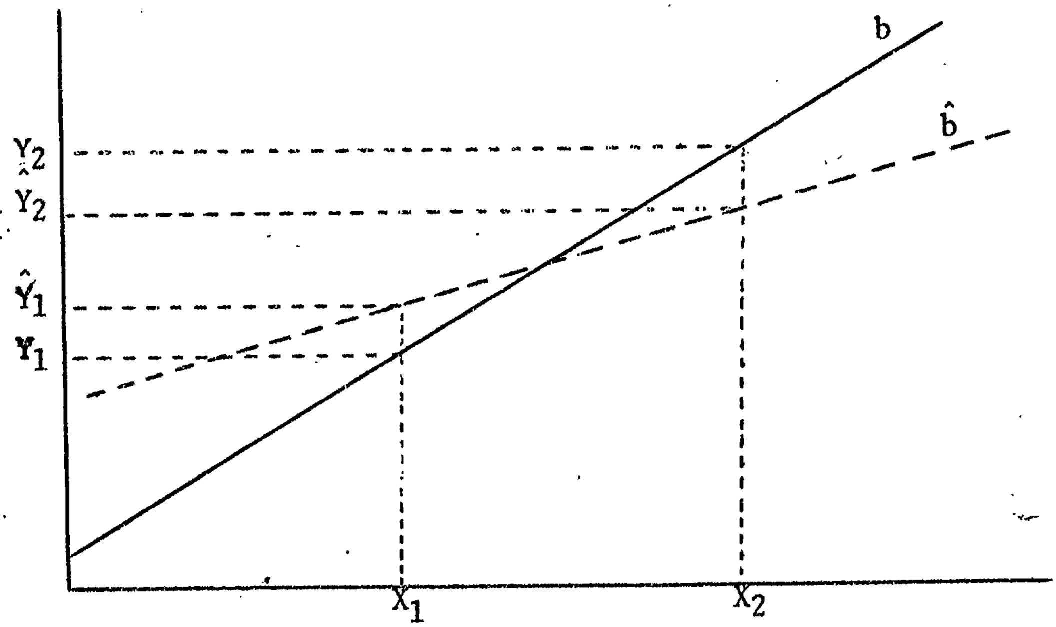
X' = Average number of kernels per head in clip plot

Y = Average head weight for same plot

X = Average number of kernels per head in same plot.

X' - X = e where e is difference between average number of kernels per head in the clip unit and in the harvest unit.

Figure 2.--The true relationship (b) compared to a downward biased estimate ($\hat{b}$).



This is an unbiased estimate of b if the X's are measured without appreciable error. When X is measured with error, the denominator becomes the Var (X + e) which equals the Var (X) + Var (e) if X and e are independent. The expected value of the covariance term works out to the same whether X is measured with or without error, provided the e_i's are independent of X and Y. The estimate of b ($\hat{b}$) will always be biased downward if the X's include any error due to either sampling or measurement techniques.

The amount of bias in $\hat{b}$ then depends upon the relative sizes of Var (X) and of Var (e).

$$\hat{b} = \frac{\text{Cov (XY)}}{\text{Var (X) + Var (e)}} < \frac{\text{Cov (XY)}}{\text{Var (X)}} = b$$

For years with below average observations an overestimate would occur with the opposite occurring in above average conditions. This increases the forecast error considerably and can influence the forecast weight of grain per head materially.

In 1967, field data were collected so the Var (e) could be estimated. Table 6 shows the estimates of the variances of X and of e, and the effects on the estimates of the regression coefficient.

The data in Table 6 show that the between unit variation (σ_e^2) are substantial and seriously affect the estimates of b. The b_1's given in the table are biased downward considerably by the σ_e^2 component of variablility.

RECOMMENDATIONS

The primary emphasis in future research should be in determining methods of estimating the between plot variances, Var (e), for the different variables used in the wheat forecasting model, for the purpose of producing relatively unbiased regression coefficients.

While the class of environmental factors (soil characteristics) studied here were not particularly well correlated with the weight of grain per head, we did not make any attempt to use a second class of environmental factors, weather data. The attempted use of weather data in predicting yields is not new, but there are more powerful tools of analysis now than were available to earlier researchers. This may justify examining these factors.

Another area of possible research could be developing estimates which are composites not only of several indications from a single probability sample, but also from consecutive monthly probability surveys during the growing season.

Table 6.--Comparisons of total variances in X (σ_X^2) with within plot variances (σ_e^2), and biased estimates of b($\hat{b}_1$) with unbiased estimates of b($\hat{b}_2$), Oklahoma, 1967.

Variable	Item	Week 1	Week 2	Week 3	Week 4
Length of head	σ_X^2	.117	.080	.101	.108
	σ_e^2	.029	.036	.023	.026
	$\hat{b}_1$	.227	.264	.116	.106
	$\hat{b}_2$	.302	.479	.150	.139
Weight of unthreshed head	σ_X^2	.048	.053	.038	.068
	σ_e^2	.025	.046	.025	.046
	$\hat{b}_1$	.237	.189	.529	.249
	$\hat{b}_2$	.489	1.434	1.546	.770
Number of fertile spiklets per head	σ_X^2	2.877	1.932	2.388	2.595
	σ_e^2	1.217	.800	.599	.684
	$\hat{b}_1$	.026	.068	.044	.053
	$\hat{b}_2$	.046	.116	.058	.072
Number of kernels per head	σ_X^2	16.064	9.373	7.774	11.897
	σ_e^2	4.454	6.183	5.418	4.536
	$\hat{b}_1$	.018	.013	.032	.019
	$\hat{b}_2$	.025	.124	.092	.030